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PROBLEMS OF DESIGNING STATE FEEDBACK CONTROLLERS FOR OBJECTS WITH TRANSFER FUNCTION ZEROS

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Abstract: The analysis of the influence of transfer function zeros on the parameters of state feedback controllers has been conducted. If a transfer function of a control object has zeros which are located closely to poles, the control object tends to singularity, and the influence of the input control signal to the states of the control object becomes weaker. The problem of the state feedback controller synthesis becomes ill-conditioned, which leads to the appearance of extremely large state feedback coefficients. In this case, the state feedback coefficients are sensitive to the parameters of the control object. As a result, the parametric robustness of the control system is reduced. Known methods of structural analysis of control object models are included amongst different methods of the numerical evaluation of the controllability and the observability, as well as methods of the model order reduction. These methods have some disadvantages, such as dependence on the state space representation form of the control object, ignoring a part of the control object model. In this paper, some ways of the preliminary structural analysis of the state space models of control objects have been proposed. The singular (Hankel) matrix is proposed for analyzing the properties of control object models. The singular matrix is the invariant characteristic of the control object in various state space forms and it characterizes the property of the control object completeness. As a result of the research, it was found that the coefficients of the state feedback controller are inversely proportional to the determinant of the singular matrix, and the determinant of the singular matrix is equal to the resultant of the transfer function polynomials. Thus the value of the determinant of the singular matrix depends on the location of the zeros of the transfer function. The method of the structural transformation (decomposition) of the control object for the defining the need of the reducing the order of the control object model is proposed.

Keywords: *zeros of a transfer function, observability, controllability, Hankel matrix, state feedback controller, resultant, residues*

1. INTRODUCTION

The main attention of researchers involved in the designing of the state feedback controllers has traditionally focused on the analysis of the eigenvalues placement of the

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control object and the choice of the desired eigenvalues placement for the closed loop system which can provide the required quality of the control system [1–3]. However, ignoring the presence of transfer function zeros in the model of the control object can lead to undesirable consequences and can negatively affect parameters of the designed state feedback controller. This fact, in particular, drew the attention of specialists working with direct drive servo systems. Using direct drive servo motors instead of gear reducers caused a significant influence of the elastic properties of the control objects on the properties of the drive system [3–5]. Elastic constructions are the systems with distributed parameters, which models have the form of partial differential equations. In practice, mathematical models of such objects are approximated to the form of ordinary differential equations [3–6]. The feature of finite-dimensional models of objects with elastic couplings is that they have transfer function zeros. The synthesis of state feedback controllers for such objects often leads to the calculation of excessively high and sensitive to object's parameters absolute values of the state feedback coefficients, which leads to decreasing of the parametric robustness of the control system [7]. Control systems with high values of the state feedback coefficients can become unstable under the noise in measurements and observations of the state vector or under the changing of the object's parameters during the operation. So, the preliminary structural analysis of state space models of control objects is needed, taking into account the location of transfer function zeros.

Consider the state space model of the direct drive servo system with two mass load with the encoder on the first mass and the state vector $\mathbf{x} = \begin{bmatrix} i & \omega_1 & \delta & \omega_2 \end{bmatrix}^T$, where i is the current of the motor, ω_1 – angular velocity of the first mass, which can be calculated from the indications of the encoder, δ – torsion angle between the first mass angle and the second mass angle, ω_2 – angular velocity of the second mass. Matrices of the state space model have the following form:

$$\mathbf{A} = \begin{bmatrix} -\dfrac{R}{L} & -\dfrac{C_e}{L} & 0 & 0 \\ \dfrac{C_m}{J_1} & -\dfrac{k_d}{J_1} & -\dfrac{C_{12}}{J_1} & \dfrac{k_d}{J_1} \\ 0 & 1 & 0 & -1 \\ 0 & \dfrac{k_d}{J_2} & \dfrac{C_{12}}{J_2} & -\dfrac{k_d}{J_2} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \dfrac{1}{L} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{C} = [0 \quad 1 \quad 0 \quad 0]$$

where R – the resistance of the phase of the motor, L – the inductance of the phase of the motor, C_e – the back-emf constant of the motor, C_m – the torque constant of the motor, J_1 – inertia of the first mass, k_d – viscosity coefficient of the elastic coupling, C_{12} – rigidity coefficient of the elastic coupling, J_2 – inertia of the second mass.

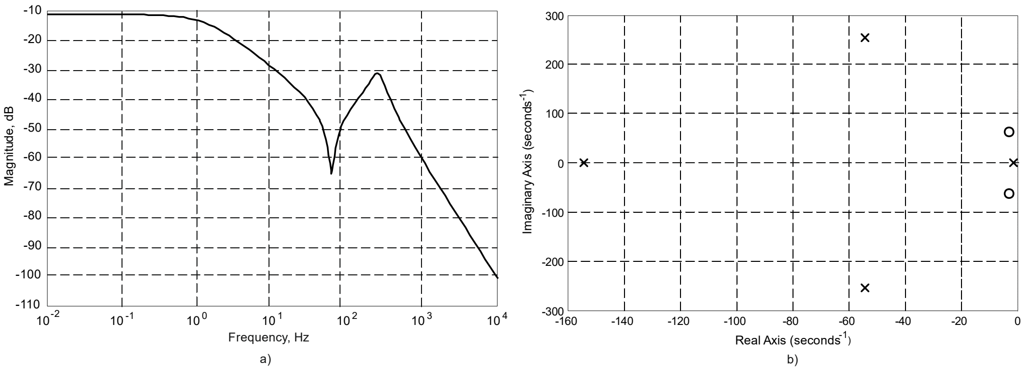

Fig. 1. Typical characteristics of the control object (1):
a) magnitude frequency characteristic, b) pole-zero map

Figure 1 shows a typical magnitude frequency characteristic and pole-zero map for the such kind of control objects. It has two complex zeros located near the poles. Distance between these poles and zeros is determined by the values of k_d, C_{12}, J_1, J_2. Sometimes control systems with state feedback controllers designed using the full state space model have insufficient quality or become unstable because of high absolute values of the state feedback coefficients. Preliminary structural analysis of such models is needed to estimate the controllability and the observability of the control object, to find out the necessity of model reduction, to make the decision about the type and structure of the controller. It is supposed that the state feedback coefficients become higher when the zero approaches to poles.

Problems of the structural analysis of mathematical models in the designing of state feedback controllers are considered in [6–17]. In [6, 8], it is suggested to use the modal dominance measures which are introduced in these works for assigning the desired eigenvalues of the closed loop system. The possibility of using some numerical measures of the controllability and the observability for choosing desired eigenvalues of the closed loop system is discussed in [9–12]. Suggested numerical measures are based on the use of some numerical characteristics of controllability and observability matrices, or controllability and observability Gramians. In [12, 13], it is suggested to change the state space representation of the mathematical model of the object, using some properties of controllability and observability Gramians. The use of modal dominance measures as well as various numerical measures of the controllability and the observability often does not provide positive results, because of their dependence on the choice of the state space representation and ignoring zeros of the transfer function.

Significant spread in the designing of state feedback controllers was obtained by the procedure of the reducing the order of the control object model [13–15]. The reduction of the order of the control object model simplifies the process of designing the state feedback controller. However, if the synthesis of state feedback controllers is based on the reduced model, it becomes necessary to conduct additional researches of the control system properties with the reduced-order controller with possible correction of the state feedback controller parameters.

One of possible solutions is the constructive development of the control object, for example, by increasing the stiffness of the mechanical part. In this case, it is possible to achieve an increase of the distance between zeros and poles of the transfer function. The methods of forming the desired spectrum of transfer zeros are considered in [16], these methods also require constructive changes of the control object at the design stage. However, in practice this approach is not always possible.

The research is devoted to the analysis of the influence of the location of transfer function zeros on the values of the state feedback coefficients and also to the development of some new ways of preliminary structural analysis of control objects. The singular matrix which is the invariant characteristic of the control object in different state space representations is introduced. The connection between the determinant of the singular matrix and the state feedback is defined. The dependence of the determinant of the singular matrix on the relative location of zeros and poles of the transfer function is proved. The numerical example is given for the simplest control object. The method of structural transformation (decomposition) of the control object which can be used in the designing of the state feedback controllers for objects with zeros of the transfer function for the defining the need of the reducing the order of the control object model is proposed.

2. CONTROLLABILITY AND OBSERVABILITY OF OBJECTS WITH TRANSFER FUNCTION ZEROS

Controllability and observability matrices of linear objects

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \\ y = \mathbf{C}\mathbf{x} \end{cases} \tag{1}$$

with a single input and a single output (SISO), according to [17] are equal to

$$\mathbf{P}_c = \begin{bmatrix} \mathbf{B} & \mathbf{AB} & \dots & \mathbf{A}^{n-1}\mathbf{B} \end{bmatrix}, \ \mathbf{P}_o = \begin{bmatrix} \mathbf{C} & \mathbf{CA} & \dots & \mathbf{CA}^{n-1} \end{bmatrix}^T$$

The transformation of the state space form according to the equation $\tilde{\mathbf{x}} = \mathbf{Mx}$ is performed in the state space of the object (1). Matrices of the new state space form of the object have the following forms:

$$\tilde{\mathbf{A}} = \mathbf{MAM}^{-1}, \ \ \tilde{\mathbf{B}} = \mathbf{MB}, \ \ \tilde{\mathbf{C}} = \mathbf{CM}^{-1}$$

Controllability and observability matrices in the new state space form

$$\tilde{\mathbf{P}}_c = \mathbf{MP}_c, \ \ \tilde{\mathbf{P}}_o = \mathbf{P}_o\mathbf{M}^{-1}, \tag{2}$$

Choosing different state space representations, different properties of controllability and observability matrices can be obtained. It follows that using expressions object controllability or object observability is not entirely correct. It will be more correct to consider the controllability or the observability of the pair of matrices (**A**, **B**) or (**A**, **C**) Thus it can be concluded that numerical measures of the controllability and the observability can be used for the synthesis of state feedback controllers only in certain cases.

Instead of the controllability and the observability it is advisable to consider some invariant property of the object that does not depend on the choice of the state space representation. The characteristic of such a property can be the matrix which has the following form:

$$\mathbf{P}_{oc} = \mathbf{P}_o \mathbf{P}_c \tag{3}$$

In accordance with (2)

$$\tilde{\mathbf{P}}_{oc} = \tilde{\mathbf{P}}_o \tilde{\mathbf{P}}_c = \mathbf{P}_o \mathbf{M}^{-1} \mathbf{M} \mathbf{P}_c = \mathbf{P}_o \mathbf{P}_c = \mathbf{P}_{oc}$$

Matrices of this structure are called Hankel matrices. This matrix is invariant characteristic of the control object in all possible state space representations. Hankel matrix has a clear physical meaning. It describes the Hankel experiment, which characterizes the connection between spaces of past input signals and future output signals of the system [6, 18]:

$$\mathbf{y} = \mathbf{P}_o \mathbf{P}_c \mathbf{u} = \mathbf{P}_{oc} \mathbf{u}$$

Hankel operator with the matrix (3), unlike the transfer function, not only describes the connection of the space of input signals and the space of output signals, but also characterizes the degree (intensity) of the participation of each pole of the object in the power transmission from the input to the output of the object. This degree of the participation is estimated by the singular numbers of the matrix (3). The matrix (3), according to [19], is called the completeness matrix of the object. In the case when the matrix $\mathbf{P}_{oc}$ is not singular, the object is called complete. In this case, the object is completely controlled and completely observed, regardless of the state space representation. If the $\operatorname{rank}\left(\mathbf{P}_{oc}\right)$ is smaller, then the state space dimension the object becomes singular. The degree of the denominator of the transfer function becomes less than the dimension of the state space. For this reason, the matrix $\mathbf{P}_{oc}$ can also be called a singular matrix. If the determinant of the singular matrix tends to zero, the connection between input and output spaces of the object becomes weaker.

3. INFLUENCE OF THE HANKEL MATRIX ON CONTROL COEFFICIENTS

The mathematical model of the linear stationary object with a single input and a single output (SISO) (1) is considered.

The problem of designing the state feedback controller that provides desired dynamic properties of the closed loop control system is considered. For solving this problem, the principle of separating procedures for calculating modal control and observing coordinates of the state vector of the object (1) is used.

The modal controller synthesis is most convenient to implement in the case when the model of the control object is presented in the controllable canonical form:

$$\begin{cases} \dot{\mathbf{x}}_c = \mathbf{A}_c \mathbf{x}_c + \mathbf{B}_c u \\ y = \mathbf{C}_c \mathbf{x}_c \end{cases} \tag{4}$$

The transfer function of the object (1) is a linear fractional function of the complex variable:

$$W(s) = \mathbf{C}\left(s\mathbf{I} - \mathbf{A}\right)^{-1}\mathbf{B} = \frac{y(s)}{u(s)} = \frac{b_{n-1}s^{n-1} + \ ... + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \ ... + a_1 s + a_0} \tag{5}$$

Matrices characterizing the model of the control object in the controllable canonical form (4), in accordance with the form of the transfer function (5), are expressed as follows:

$$\mathbf{A}_c = \begin{bmatrix} 0 & 1 & 0 & ... & 0 \\ 0 & 0 & 1 & ... & 0 \\ ... & ... & ... & ... & 0 \\ 0 & 0 & 0 & ... & 1 \\ -a_0 & -a_1 & -a_2 & ... & -a_{n-1} \end{bmatrix}, \quad \mathbf{B}_c = \begin{bmatrix} 0 \\ 0 \\ ... \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{C}_c = \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_1 \\ b_{n-1} \end{bmatrix}^T \tag{6}$$

The representation of the model in the controllable canonical form ensures the existence of the control algorithm that provides the desired assignment of eigenvalues of the characteristic polynomial of the closed loop system [6]. The control law for the object (1) can be written as follows:

$$u = -\mathbf{K}_c \mathbf{x}_c \tag{7}$$

The coefficients of the matrix $\mathbf{K}_c$ in the controllable canonical form are calculated as follows:

$$\mathbf{K}_c = \mathbf{a}_{cg} - \mathbf{a}_c \tag{8}$$

where the matrix $\mathbf{a}_{cg}$ is the row of coefficients of the desired characteristic polynomial of the closed loop system, and the matrix $\mathbf{a}_c$ is the row of coefficients of the characteristic polynomial of the matrix $\mathbf{A}_c$.

The state vectors of models (1) and (4) are related by the transformation matrix

$$\mathbf{x}_c = \mathbf{M}_c \mathbf{x} \tag{9}$$

wherein:

$$\mathbf{A}_c = \mathbf{M}_c \mathbf{A} \mathbf{M}_c^{-1}, \quad \mathbf{B}_c = \mathbf{M}_c \mathbf{B}, \quad \mathbf{C}_c = \mathbf{C} \mathbf{M}_c^{-1} \tag{10}$$

The transformation matrix $\mathbf{M}_c$ can be represented using controllability matrices in different state space forms.

The controllability matrix in the state space form (1) is:

$$\mathbf{P}_c = \begin{bmatrix} \mathbf{B} & \mathbf{AB} & \dots & \mathbf{A}^{n-1}\mathbf{B} \end{bmatrix}$$

The controllability matrix in the state space form (4) is following:

$$\tilde{\mathbf{P}}_\mathbf{c} = \begin{bmatrix} \mathbf{B}_c & \mathbf{A}_c\mathbf{B}_c & \dots & \mathbf{A}_c^{n-1}\mathbf{B}_c \end{bmatrix}$$

Using the equation (10) the following relation can be obtained:

$$\tilde{\mathbf{P}}_c = \mathbf{M}_c \mathbf{P}_c$$

Controllability matrices are known in both forms, so the transformation matrix can be represented as follows:

$$\mathbf{M}_c = \tilde{\mathbf{P}}_c \mathbf{P}_c^{-1} \tag{11}$$

Using Eqs. (9) and (11), the control law (7) can be rewritten in the following form:

$$u = -\mathbf{K}_c \tilde{\mathbf{P}}_c \mathbf{P}_c^{-1} \mathbf{x} \tag{12}$$

In most practical cases, some of the coordinates of the state vector $\mathbf{x}$ of the object (1) are not measured, so it becomes necessary to design the observing device. The representation of the model in the observable canonical form ensures the existence of the algorithm that performs the estimation of the state vector of the object. Using the Kalman duality principle [17], matrices of the observable canonical form are chosen as follows:

$$\mathbf{A}_o = \mathbf{A}_c^T = \begin{bmatrix} 0 & 0 & \dots & 0 & -a_0 \\ 1 & 0 & \dots & 0 & -a_1 \\ 0 & 1 & \dots & 0 & -a_2 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 & -a_{n-1} \end{bmatrix}, \quad \mathbf{B}_o = \mathbf{C}_c^T, \quad \mathbf{C}_o = \mathbf{B}_c^T \tag{13}$$

The mathematical model of the state observer in the observable canonical form can be written as follows:

$$\begin{cases} \dot{\tilde{\mathbf{x}}}_o = \mathbf{A}_o \tilde{\mathbf{x}}_o + \mathbf{B}_o u + \mathbf{L}_o (y - \mathbf{C}_o \tilde{\mathbf{x}}_o) \\ y = \mathbf{C}_o \tilde{\mathbf{x}}_o \end{cases} \tag{14}$$

In accordance with the structure of matrices $\mathbf{A}_o$, $\mathbf{C}_o$, the matrix of the observer input by the observer error has the following form:

$$\mathbf{L}_o = \mathbf{a}_{og}^T - \mathbf{a}_o^T$$

where the matrix $\mathbf{a}_{og}^T$ is the column of coefficients of the desired characteristic polynomial, and the matrix $\mathbf{a}_o^T$ is the column of coefficients of the characteristic polynomial of the matrix $\mathbf{A}_o$.

State vectors of models (1) and (13) are related as follows:

$$\mathbf{x}_o = \mathbf{M}_o\mathbf{x} \tag{15}$$

wherein

$$\mathbf{A}_o = \mathbf{M}_o\mathbf{A}\mathbf{M}_o^{-1}, \quad \mathbf{B}_o = \mathbf{M}_o\mathbf{B}, \quad \mathbf{C}_o = \mathbf{C}\mathbf{M}_o^{-1} \tag{16}$$

The transformation matrix $\mathbf{M}_o$ can be represented using observability matrices of the object in different state space forms.

The observability matrix in the state space form (1) is:

$$\mathbf{P}_o = \begin{bmatrix} \mathbf{C}^T & (\mathbf{CA})^T & (\mathbf{CA}^2)^T & \dots & (\mathbf{CA}^{n-1})^T \end{bmatrix}^T$$

The observability matrix in the form (13) is following:

$$\tilde{\mathbf{P}}_o = \begin{bmatrix} \mathbf{C}_o^T & (\mathbf{C}_o\mathbf{A}_o)^T & (\mathbf{C}_o\mathbf{A}_o^2)^T & \dots & (\mathbf{C}_o\mathbf{A}_o^{n-1})^T \end{bmatrix}^T$$

From relations (16) it can be written:

$$\tilde{\mathbf{P}}_o = \mathbf{P}_o\mathbf{M}_o^{-1}$$

Observability matrices are known in both forms. Therefore the transformation matrix to the observable canonical form is determined by the following expression:

$$\mathbf{M}_o = \tilde{\mathbf{P}}_o^{-1}\mathbf{P}_o \tag{17}$$

For the implementation of the control law (12), observed states $\tilde{\mathbf{x}}_o = \mathbf{M}_o\mathbf{x}$ are used. In accordance to this fact the state vector can be written as follows:

$$\mathbf{x} = \mathbf{M}_o^{-1}\tilde{\mathbf{x}}_o$$

Substituting this value into the expression (12), the following equation for the control law can be obtained:

$$u = -\mathbf{K}_c\tilde{\mathbf{P}}_c\mathbf{P}_c^{-1}\mathbf{M}_o^{-1}\tilde{\mathbf{x}}_o$$

In accordance with the expression (17), the last expression is converted to the following form:

$$u = -\mathbf{K}_c \tilde{\mathbf{P}}_c \mathbf{P}_c^{-1} \mathbf{P}_o^{-1} \tilde{\mathbf{P}}_o \tilde{\mathbf{x}}_o \;\Rightarrow\; u = -\mathbf{K}_c \tilde{\mathbf{P}}_c (\mathbf{P}_o \mathbf{P}_c)^{-1} \tilde{\mathbf{P}}_o \tilde{\mathbf{x}}_o$$

Taking into account the expression for the singular matrix (3), the last expression is rewritten in the following form:

$$u = -\mathbf{K}_c \tilde{\mathbf{P}}_c (\mathbf{P}_{oc})^{-1} \tilde{\mathbf{P}}_o \tilde{\mathbf{x}}_o \tag{18}$$

According to the expression (18), the singular matrix $\mathbf{P}_{oc}$ must be reversed. Thus, the state feedback coefficients are inversely proportional to the determinant of the singular matrix. When the determinant of this matrix tends to zero, the absolute values of the state feedback coefficients tend to infinity.

The characteristic of two polynomials connecting distances between their roots (for the transfer function – between zeros and poles) is the resultant of polynomials:

$$\operatorname{res}\left(u(s),\ y(s)\right) = \prod_{j=1}^{n-1} \left(\alpha_i - \beta_j\right)$$

where α_i are poles of the transfer function, and β_j are zeros of the transfer function.

The resultant of the transfer function polynomials is equal to the product of all distances between their zeros and poles. The equality of the determinant of the Hankel matrix of the object (1) and the resultant of transfer function (5) polynomials can be proved using the Kronecker method of calculating the resultant [20].

Thus, if the zero of the transfer function (5) approaches to any pole, the determinant of the singular matrix decreases and absolute values of the state feedback coefficients increase. Accordingly, the presence of zeros located closely to poles of the transfer function affects the values of the state feedback coefficients. An attempt to correct such poles leads to increasing of the parametric sensitivity of the control system.

4. THE METHOD OF CONTROL OBJECT DECOMPOSITION

The most often encountered in practice case when the transfer function (5) has no multiple poles is considered, taking into account that the resultant of numerator and denominator polynomials is equal to the determinant of the singular matrix. Under this condition, the linear fractional transfer function can be represented by the sum of elementary fractions:

$$W(s) = \frac{r_1}{s-\alpha_1} + \frac{r_2}{s-\alpha_2} + \ldots + \frac{r_n}{s-\alpha_n} \tag{19}$$

where α_i are poles of the transfer function (5), including complex conjugate poles. According to Eq. (19), the numerators of elementary fractions can be calculated as follows:

$$r_i = \lim_{s \to \alpha_i} (s - \alpha_i) W(s) \tag{20}$$

The value r_i calculated from (20) is called the residue of the function $W(s)$ at the pole α_i. Performing calculations according to Eq. (20), we obtain:

$$r_i = \frac{b_{n-1} \prod_{j=1}^{n-1} \left(\alpha_i - \beta_j \right)}{\prod_{\substack{j=1 \\ j \neq i}}^{n-1} \left(\alpha_i - \alpha_j \right)},$$

where α_i are poles of the transfer function, and β_j are zeros of the transfer function.

The absolute value of the numerator of the residue at the pole α_i is proportional to the product of distances between this pole and all zeros of the transfer function. The absolute value of the denominator of the residue at the pole is equal to the product of distances between this pole and other poles of the transfer function. If the pole is equal to some zero of the transfer function, the value of the residue at this pole becomes zero. So, if the value of the pole approaches to the value of any zero of the transfer function, absolute value of the corresponding residue becomes smaller. In the structural diagram, the sum of transfer functions corresponds to the parallel connection of corresponding structural elements. For this reason, the residue at the pole is the transmission coefficient of the corresponding branch of the structural diagram. If the absolute value of the corresponding residue becomes smaller, the part of the power transmitted by the corresponding branch of the structural diagram from the object's input to its output becomes smaller as well. So, attempts to change the value of this pole using the state feedback controllers lead to the appearance of excessively high state feedback coefficients.

The structural diagram of the object (1) with the transfer function (5) is represented in the form shown in Fig. 3.

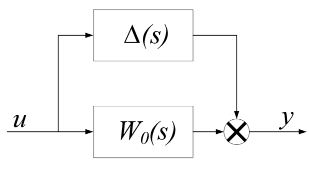

Fig. 2. Decomposed diagram of the control object

In this interpretation, the structural element with the transfer function $\Delta(s)$ can be considered as the multiplicative uncertainty for the object with the transfer function $W_o(s)$.

The sum of absolute values of residues which related to the transfer function $\Delta(s)$ is defined as m_Δ, and the sum of absolute values of all residues of the series (19) is defined as $m_\Sigma = \sum_{i=1}^{n} |r_i|$. The choice of $\Delta(s)$ should be carried out using the following condition:

$$\frac{m_\Delta}{m_\Sigma} \le \delta$$

where the number δ characterizes the part of the transmitting power, which has a little effect on the overall behavior of the object when the control signal is applied. The value of this number has to be selected from operating conditions of the object and requirements for the designed control system.

The representation of the control object in the form shown in Fig. 1 is typical of the theory of robust control. Methods of the robust control designing, including those based on the MATLAB software package, were considered in [21, 22].

Suggested method of the control object decomposition can be used for defining the need of the reducing the order of the control object model. Residues of the transfer function can be used as numerical characteristics of this procedure. The method requires experimental verification on a large scale of real objects with different parameters to find out some dependences and regularities in procedures of decomposition and reduction of the control object. It should be noted that the more detailed research devoted to the state feedback controller designing for proposed representation of the control object is needed.

5. NUMERICAL EXAMPLE

Consider the model of the linear control object with the following transfer function:

$$W(s) = \frac{s+c}{(s+a)(s+b)} \tag{21}$$

The control object is represented in three state space canonical forms: diagonal, controllable, and observable, then controllability and observability matrices are composed for each form of the object representation.

Residues at the poles of the object (21):

$$r_a = \frac{c-a}{b-a}, \quad r_b = \frac{c-b}{a-b}$$

The diagonal canonical form:

$$\mathbf{A}=\begin{bmatrix}-a & 0\\ 0 & -b\end{bmatrix},\quad \mathbf{B}=\begin{bmatrix}r_a\\ r_b\end{bmatrix},\quad \mathbf{C}=\begin{bmatrix}1 & 1\end{bmatrix}$$

$$\mathbf{P}_c=\begin{bmatrix}\dfrac{a-c}{a-b} & \dfrac{a(c-a)}{a-b}\\ \dfrac{c-b}{a-b} & \dfrac{b(b-c)}{a-b}\end{bmatrix},\quad \mathbf{P}_o=\begin{bmatrix}1 & 1\\ -a & -b\end{bmatrix}$$

The controllable canonical form:

$$\mathbf{A}_c=\begin{bmatrix}0 & 1\\ -ab & -(a+b)\end{bmatrix},\quad \mathbf{B}_c=\begin{bmatrix}0\\ 1\end{bmatrix},\quad \mathbf{C}_c=\begin{bmatrix}c & 1\end{bmatrix}$$

$$\mathbf{P}_{c1}=\begin{bmatrix}0 & 1\\ 1 & -a-b\end{bmatrix},\quad \mathbf{P}_{o1}=\begin{bmatrix}c & 1\\ -ab & c-b-a\end{bmatrix}$$

The observable canonical form:

$$\mathbf{A}_o=\begin{bmatrix}0 & -ab\\ 1 & -(a+b)\end{bmatrix},\quad \mathbf{B}_o=\begin{bmatrix}c\\ 1\end{bmatrix},\quad \mathbf{C}_o=\begin{bmatrix}0 & 1\end{bmatrix}$$

$$\mathbf{P}_{c2}=\begin{bmatrix}c & -ab\\ 1 & c-b-a\end{bmatrix},\quad \mathbf{P}_{o2}=\begin{bmatrix}0 & 1\\ 1 & -a-b\end{bmatrix}$$

The singular matrix of the control object is composed. The determinant of the singular matrix, and also the resultant of the transfer function polynomials are calculated:

$$\mathbf{P}_{oc}=\begin{bmatrix}1 & -(a+b-c)\\ -(a+b-c) & a^2+ab-ca+b^2-cb\end{bmatrix}$$

$$\det(\mathbf{P}_{oc})=(c-b)(a-c),\qquad \operatorname{res}(W(s))=(b-c)(a-c)$$

It is easy to verify that the singular matrix is the same in all the canonical forms which have been considered. Also, it is obvious that the determinant of the singular matrix and the resultant of the polynomials are equal up to a sign.

Let the desired characteristic polynomial of a closed loop system be as follows:

$$D_g=s^2+a_1s+a_2$$

Suppose that the state observer, which restores the state variables of the object, is designed. The state feedback coefficients for observer states are calculated in accordance with Eqs. (8) and (18):

$$K_U(1) = \frac{a_1c + ab - ac - bc - a_2}{(c-b)(a-c)}$$

$$K_U(2) = \frac{-a_1ac + a_1ab + a_2c - a_1bc - ab^2 - a^2b + a^2c + b^2c + abc)}{(c-b)(a-c)}$$

It follows from above equations that the values of the state feedback coefficients are inversely proportional to the determinant of the singular matrix.

The numerical simulation of the system using following parameters is performed: $a = 25$, $b = 15$, $a_1 = 100$, $a_2 = 2400$, $c = 10 \ldots 30$. Desired eigenvalues of the closed loop system are [–40, –60].

Figure 3 shows the dependence of the absolute value of the determinant of the singular matrix on the location of the object's zero. Figure 4 shows the dependence of absolute values of the state feedback coefficients on the location of the object's zero.

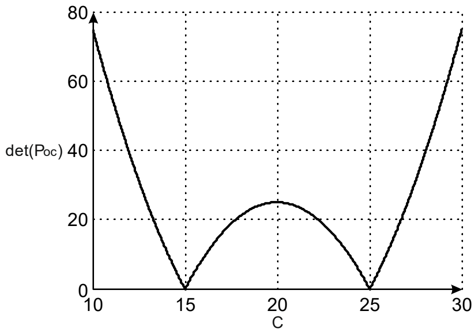

Fig. 3. Absolute value of the determinant of the singular matrix

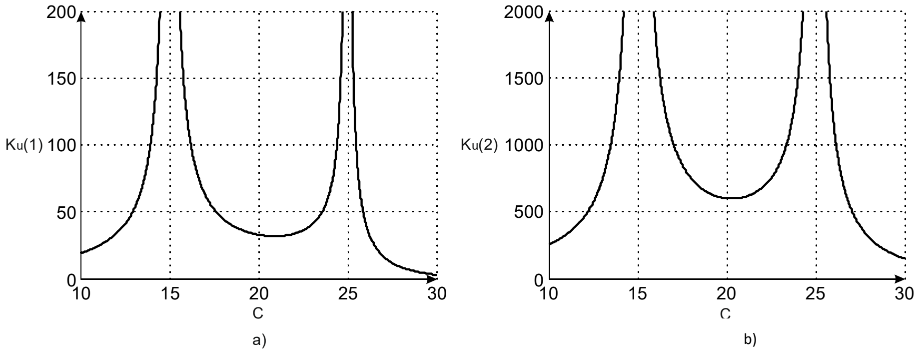

Fig. 4. Absolute values of the state feedback coefficients:
a) coefficient of the first state $K_U(1)$, b) coefficient of the second state $K_U(2)$

Submitted dependences clearly demonstrate that the state feedback coefficients are inversely proportional to the determinant of the singular matrix. The determinant of the singular matrix becomes smaller and the state feedback coefficients become higher when zero approaches to poles. Closed loop control systems with such high values of the state feedback coefficients can be unstable under the noise in measurements and observations of the state vector or under the changing of the object's parameters during the operation. So these dependences also demonstrate the negative influence of the transfer function zeros on controller parameters. Figure 5 shows the dependence of absolute values of residues on the location of the zero.

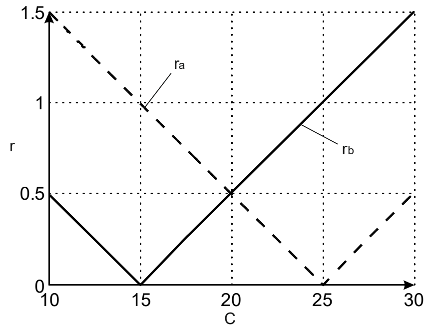

Fig. 5. Absolute values of the residues of the control object

Obviously, if the pole approaches to zero, the value of the corresponding residue decreases. This means that the part of the power transmitted to the output of the system by the subspace corresponding to this pole decreases as well.

6. CONCLUSIONS

Zeros of the transfer function can negatively affect parameters of the state feedback controllers. Some ways of preliminary structural analysis of state space models were considered. Singular (Hankel) matrix of the control object was introduced. Influence of the singular matrix on the process of controller designing was analytically researched. There was found out that the state feedback coefficients are inversely proportional to the determinant of the singular matrix of the control object. Also the determinant of the singular matrix is equal to the product of all distances between poles and zeros of the transfer function, which is called the resultant of transfer function polynomials. So, the attempt to replace the pole located closely to any zero of transfer function entails the excessive growth of absolute values of the state feedback coefficients. On the other hand, such a pole has a little effect on the connection between state spaces of input and output signals. In this case, it is advisable to decompose the control object by isolating poles located near zeros as the transfer function of the multiplicative uncertainty of the control object. Such poles can be defined by comparing absolute values of residues at all transfer function poles. Suggested representation of the control object can be used for designing the robust controller or reducing the order of the state space model. Represented simple numerical example clearly demonstrates analytical dependences which were found out during the research. Suggested ways of preliminary structural analysis of state space models requires experimental verification on a large scale of real objects with different parameters.

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