DIRECT FIELD-ORIENTED CONTROL
OF SIX-PHASE INDUCTION MOTOR
WITH FUZZY-LOGIC SPEED CONTROLLER*

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Abstract: The paper presents a Direct Field-Oriented Control (DFOC) of six-phase induction motor with Fuzzy-Logic speed controller. Mathematical models of the six-phase squirrel-cage induction motor and the six-phase voltage source inverter have been presented. A method of space vector modulation used to control a six-phase voltage source inverter has been described. Simulation studies of the DFOC with Fuzzy-Logic speed controller have been carried out and the results presented and discussed.

Keywords: six-phase induction motor, Direct Field-Oriented Control, Space Vector Modulation, fuzzy-logic control

1. INTRODUCTION

The development of frequency converter systems provides the possibility of industrial applications of multi-phase induction motors (the motors with the number of phases greater than three). Power electronic frequency converters can be built as multi-phase inverters with powering from the three phase mains. Multi-phase induction motors have a lot of advantages in comparison with conventional three-phase induction motors [5], [6], [9], [10]. The nominal values of the stator phase currents of multi-phase induction motors are smaller than nominal values of the phase currents in three-phase induction motors. This allows the requirements for ampacity of the motor and inverter semiconductor elements to be reduced. Other beneficial properties include: reduction of the ripples of electromagnetic torque and the losses caused by current higher harmonics in the dc link of frequency converter. Multiphase induction motors allow greater reliability of the drive systems to be ensured because they may be conditionally operated at failure of one or more stator phases [5], [6], [9], [10].
In the literature there are some publications about the control methods of the multi-phase induction motors. Most studies are based on the Field-Oriented Control and Direct Torque Control with PI controllers [5], [6], [9]. A small number of publications address the problems of the Fuzzy-Logic control of the six-phase induction motors, but only with the use of the PWM modulation. In this paper, an analysis of Direct Field-Oriented Control (DFOC) of six-phase induction motor with Space Vector Modulator and Fuzzy-Logic speed controller with scaling factors chosen with the use of genetic algorithms has been presented and discussed. At present the Fuzzy-Logic controllers are applied in modern control systems because they make it possible to achieve good accuracy of control, robustness and ability to work in noise condition [1]–[4], [8].

2. MATHEMATICAL MODEL OF SIX-PHASE INDUCTION MOTOR

In this paper, a symmetrical six-phase squirrel-cage induction motor with the axes of stator windings shifted electrically by sixty degrees is considered. A mathematical model of the six-phase squirrel-cage induction motor has been formulated on the basis of commonly used simplifying assumptions presented in detail in [5], [6], [9], [10]. The model of the motor considered in the system of phase coordinates is described by differential equations with coefficients changing as a function of the rotation angle of the rotor. The analysis of this type of model is difficult. The phase variables of six-phase induction motor can be converted to new systems of transformed variables, considered in common for stator and rotor reference frames. The mathematical model of six-phase motor formulated for transformed variables is described by differential equations with constant coefficients [5], [6], [9].

The general equations of six-phase induction motor after transformations take the following form [5], [6], [9]:

- the voltage equations of the stator and rotor in the rectangular \( x-y \) coordinate system, which rotates relative to the stator at arbitrary angular speed \( \omega_k \)

\[
\begin{align*}
    u_{sx} &= R_s i_{sx} - \omega_k \psi_{sy} + p \psi_{sx} , \\
    u_{sy} &= R_s i_{sy} + \omega_k \psi_{sx} + p \psi_{sy} , \\
    0 &= R_r i_{rx} - (\omega_k - \omega_e) \psi_{ry} + p \psi_{rx} , \\
    0 &= R_r i_{ry} + (\omega_k - \omega_e) \psi_{rx} + p \psi_{ry} \\
\end{align*}
\]  

(1)

- the stator voltage equations in the additional coordinate system \( z_1-z_2 \)

\[
\begin{align*}
    u_{sz_1} &= R_s i_{sz_1} + p \psi_{sz_1} , \\
    u_{sz_2} &= R_s i_{sz_2} + p \psi_{sz_2} , \\
\end{align*}
\]  

(3)

- equation of the electromagnetic torque

\[
T_e = \frac{6}{2} p_s (\psi_{ry} i_{sx} - \psi_{rx} i_{sy}) ,
\]

(4)

(2)
– mechanical motion equation

\[ T_e - T_m = \frac{J_m}{p_b} \cdot p \omega_e , \]  

(5)

where: \( u_{sx}, u_{sy} \) – components of the stator voltage vectors in the \( x-y \) coordinate system; \( u_{sz1}, u_{sz2} \) – components of the stator voltage vectors in the additional \( z1-z2 \) coordinate system; \( i_{sx}, i_{sy}, i_{rx}, i_{ry} \) – components of the stator and rotor current vectors in the \( x-y \) coordinate system; \( i_{sz1}, i_{sz2} \) – components of the stator current vector in the additional \( z1-z2 \) coordinate system; \( \psi_{sx}, \psi_{sy}, \psi_{rx}, \psi_{ry} \) – components of the stator and rotor flux linkage vectors in the \( x-y \) coordinate system; \( \psi_{sz1}, \psi_{sz2} \) – components of the stator flux linkage vectors in the additional \( z1-z2 \) coordinate system; \( \omega_k \) – arbitrary angular speed of the coordinate system relative to the stator; \( \omega_e \) – the electrical angular speed of the motor; \( T_e \) – the motor electromagnetic torque; \( T_m \) – the load torque; \( J_m \) – the motion inertia of the drive system; \( R_s, R_r \) – stator and rotor phase resistance; \( p_b \) – the number of motor pole pairs; \( p = \frac{d}{dt} \) – derivative operator.

In the analysis, the star connection of six-phase stator winding with single isolated neutral point and short-circuited circuits of equivalent six-phase rotor winding are assumed. For this reason the equations for stator and rotor zero sequence components and for all rotor additional components are neglected in the analysis because these variables are identically equal to zero.

On the basis of equation (4) it can be stated that in the multi-phase induction motors only state variables determined in the general coordinate system \( x-y \) (the stationary \( \alpha-\beta \) coordinate system in the case \( \omega_k = 0 \)) are involved in the conversion of electromechanical energy and in generation of the electromagnetic torque [5], [9], [10]. The state variables considered in the additional systems labeled with the index \( z \) (described by equation (3)) are not involved in the generation of the electromagnetic torque, but they can cause enlargement of the amplitudes of the stator phase currents and power losses in the stator windings. For these reasons these variables should also be included in the precise analysis of multi-phase motor.

3. MATHEMATICAL MODEL OF SIX-PHASE VOLTAGE SOURCE INVERTER

A scheme of the six-phase voltage source inverter (VSI) with six-phase induction motor is shown in Fig. 1.

It is considered that the six-phase induction motor is supplied by the two-level six-phase voltage source inverter. Each leg of VSI consists of two power electronics switches operated alternately. Assuming an ideal operation of the VSI switches, each branch of the inverter can be presented as a two-state switch.
Voltage space vectors generated by the six-phase VSI in the stationary \((\omega_k = 0)\) rectangular coordinate system \(\alpha-\beta\) and in the \(z_1-z_2\) additional coordinate system can be determined in the general form [5], [10]

\[
    u_{s\alpha\beta} = u_{s\alpha} + j u_{s\beta} = \frac{2}{6} \left( S_1 + a S_2 + a^2 S_3 + a^3 S_4 + a^4 S_5 + a^5 S_6 \right) \cdot u_d,
\]

\[
    u_{s_{z1z2}} = u_{s1} + j u_{s2} = \frac{2}{6} \left( S_1 + a^2 S_2 + a^4 S_3 + a^6 S_4 + a^2 S_5 + a^4 S_6 \right) \cdot u_d,
\]

where: \(a = \exp(j2\pi/6)\); \(S_1, \ldots, S_6\) – the switching functions of the six-phase VSI switches \((S_i = 0 \text{ or } S_i = 1, i = 1, \ldots, 6)\) and \(u_d\) – the voltage in the DC link of the VSI.

Voltage space vectors in the system \(\alpha-\beta\) are presented in Fig. 2a and in the additional system \(z_1-z_2\) are presented in Fig. 2b.
For the six-phase voltage source inverter, the total number of state combinations of inverter switches is equal to $2^6 = 64$. These 64 space voltage vectors divide the switching pattern into 12 sectors. In the set of all generated voltage space vectors the active and zero vectors can be identified. In Fig. 2, the voltage space vectors for six-phase voltage source inverter have been specified by the decimal numbers. These decimal numbers can be converted into 6-position binary numbers. The binary bits obtained determine the states of the switches in the individual legs of six-phase voltage source inverter.

4. SPACE VECTOR MODULATION METHOD

The Space Vector Modulation (SVM) method has been implemented for the control of six-phase voltage source inverter with the six-phase induction motor. Due to the great number of voltage space vectors there are some differences in application of the space vector modulation algorithm for six-phase voltage source inverter in comparison with modulation for conventional three-phase voltage source inverters.

In the space vector modulation technique presented the reference voltage vector is synthesized by using the appropriate switching times only of two long voltage vectors situated in the same sector in which the reference voltage vector is located and additionally two zero voltage vectors [10]. A graphical interpretation of the applied SVM method for six-phase voltage source inverter is presented in Fig. 3. For the case where the reference voltage vector $\mathbf{u}_{s_{\text{ref}}}$ falls into Sector 1 it can be synthesized by choosing two long voltage vectors: $\mathbf{u}_{49}$, $\mathbf{u}_{56}$ and choosing the selected zero vector $\mathbf{u}_0$.

![Fig. 3. The principle of space vector modulation for six-phase voltage source inverter](image)

For the case presented in Fig. 3, when the reference voltage vector is situated in Sector 1 the principle of space vector modulation method can be described by the following equations

$$\mathbf{u}_{s_{\text{ref}}} \cdot T_s = \mathbf{u}_{49} \cdot t_1 + \mathbf{u}_{56} \cdot t_2 + \mathbf{u}_0 \cdot t_0, \quad (8)$$


\[ t_0 = T_s - t_1 - t_2 , \]  

where: \( t_1, t_2 \) – switching times of long voltage vectors; \( t_0 \) – switching time of zero voltage vectors; \( T_s \) – switching period; \( \Theta \) – the angle position of reference voltage vector.

5. FUZZY-LOGIC SPEED CONTROLLER

In the paper, the advanced control system of six-phase induction motor has been considered. The system has been based on DFOC control with application of the Fuzzy-Logic speed controller. A scheme of the Fuzzy-Logic speed controller considered is presented in Fig. 4 [1], [2].

![Image of the block diagram of the Fuzzy-Logic speed controller](image)

The Fuzzy-Logic speed controller presented consists of five parts [1], [2]:
- Part 1 is responsible for determination of two input control signals. The first input signal \( e \) is the difference between the reference value of the motor speed and the actual value of the motor speed. The second input signal \( \Delta e \) is the time derivative of the first input signal.
- Part 2 executes fuzzification of the input signals. The triangle-shaped membership functions have been used in this part of Fuzzy-Logic speed controller. The selected triangle-shaped membership functions are presented in Fig. 5.
The weight coefficients were chosen as: N (negative), NB (negative big), NM (negative medium), NS (negative small), NL (negative low), ZE (zero), P (positive), PL (positive low), PS (positive small), PM (positive medium), PB (positive big).

– Part 3 is doing the multiplication of the appropriate output signals from Part 2. The input signals in this part are signals of activation levels of membership functions.

– Part 4 performs multiplication of signals of the activation levels of the rules and weight coefficients. Multiplication in this part takes place in accordance with the multiplication rules. Linguistic Rule Table applied in the Fuzzy-Logic speed controller is presented in Table 1.

<table>
<thead>
<tr>
<th>Control error</th>
<th>Derivative of control error $\Delta e$</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>NB</td>
</tr>
<tr>
<td>NB</td>
<td>NB</td>
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<td>NS</td>
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<tr>
<td>PS</td>
<td>NL</td>
</tr>
<tr>
<td>PB</td>
<td>ZE</td>
</tr>
</tbody>
</table>

– Part 5 calculates the output signal from the Fuzzy-Logic speed controller. The defuzzification procedure is described by the equation

$$u(k) = \frac{\sum_{i=1}^{n} w_i a_i}{\sum_{i=1}^{n} a_i}.$$  \hspace{1cm} (10)

In the defuzzification procedure presented by equation (10) the method of the Center of Gravity (COG) has been applied [1]. The additional integral block is used at the output of the Fuzzy-Logic Controller.
The scaling factors $k_e$, $k_{de}$ and $k_{du}$ of the Fuzzy-Logic controller have been chosen with the use of genetic algorithms [4], [8]. The other elements of the structure of the Fuzzy-Logic controller were fixed. The quality index $K$ based on using integral square error (ISE) criterion has been utilized in the optimization procedure. The applied ISE criterion is presented below

$$K = \min_{0}^{\infty} (\omega_c^* - \omega_c)^2 dt. \quad (11)$$

The process of the DFOC structure optimization was carried out with the use of Optimization Tool in Matlab® Software.

6. DIRECT FIELD-ORIENTED CONTROL SYSTEM WITH FUZZY-LOGIC SPEED CONTROLLER

The system of six-phase induction motor with the Direct Field-Oriented Control and Fuzzy-Logic speed controller is shown in Fig. 6. In order to implement the DFOC control system the estimator of the instantaneous magnitude and the angle of the rotor flux vector has been used. The estimator uses the signals of measured stator phase currents and motor angular speed [7].

![Fig. 6. System of DFOC of six-phase induction motor with Fuzzy-Logic speed controller](image-url)

In the DFOC structure four control loops have been applied: the control loop for motor angular speed $\omega_e$ with Fuzzy-Logic controller, the control loop for magnitude of the rotor flux vector $\psi_r$ with PI controller and two control loops for $x$ and $y$ component of stator current vector. The $y$ component of stator current vector, responsible for
electromagnetic torque control is determined by the Fuzzy-Logic controller. The $x$ component of stator current vector, responsible for the rotor flux control is determined by the PI controller of magnitude of the rotor flux vector. The reference values of $x$ and $y$ component of stator current vector are compared with the transformed values of measured stator phase currents in two inner control loops. Reference values of the stator voltage vector components are transformed to the $\alpha$-$\beta$ coordinate system and are given to the block SVM. SVM block sets the switching states of the six-phase voltage source inverter.

7. SIMULATION RESULTS

The simulation model of the Direct Field-Oriented Control of six-phase induction motor with Fuzzy-Logic speed controller has been implemented in Matlab/Simulink® Software. Simulation studies were carried out for the six-phase squirrel-cage induction motor with the parameters: $P_N = 3$ kW, $U_{fN} = 230$ V, $f_N = 50$ Hz, $\omega_{eN} = 295$ rad/s, $p_h = 2$, $R_s = 1.9 \ \Omega$, $R_r = 2.1 \ \Omega$, $L_{ls} = L_{lr} = 0.013 \ \text{H}$, $L_m = 0.6 \ \text{H}$.

The simulation studies of DFOC of six-phase induction motor with Fuzzy-Logic speed controller were performed for the assumed trajectory of the reference speed. After reaching the set value of motor speed, the step change of the load torque has been forced during the simulation. The waveforms of the reference speed, measured speed and difference between these speeds of the six-phase induction motor are presented in Fig. 7. It can be stated that the measured speed follows the reference speed with great accuracy. The waveforms of the load and electromagnetic torque of six-phase induction motor for DFOC with Fuzzy-Logic speed controller are presented in Fig. 8. The values of electromagnetic torque depend on the working states of the drive system. It can be stated that the absolute values of electromagnetic torque are high during the states of increasing and decreasing of motor speed. The DFOC system with Fuzzy-Logic speed controller provides rapid responses of electromagnetic torque during the changes of the load torque.

![Fig. 7. The waveforms of reference speed, measured speed and difference between the speeds of six-phase induction motor for DFOC with Fuzzy-Logic speed controller](image-url)
The waveforms of the stator phase current of the six-phase induction motor for the DFOC with Fuzzy-Logic speed controller are presented in Fig. 9a. The amplitude of stator phase current depends on the working states of the drive system. The values of the current amplitude are high in dynamic states. In the steady-state operations the values of the current amplitude are smaller and are approximately constant.

The trajectory of the estimated magnitude of the rotor flux vector for DFOC with Fuzzy-Logic speed controller is presented in Fig. 9b. It can be noticed that the magnitude of the rotor flux vector is regulated at the reference nominal value.

**8. CONCLUSIONS**

The simulation studies of the drive system with six-phase induction motor confirmed the good operation and properties of the DFOC control system with Fuzzy-Logic speed controller under study.

The measured motor speed follows the reference motor speed with great accuracy. The values of the motor electromagnetic torque depend on the working states of the drive system. The absolute values of the electromagnetic torque are high during increasing and decreasing of the motor speed and during the action of load torque.
The amplitudes of stator phase currents depend on the condition of the drive system: they are high in dynamic states and they are small at the steady-state operations. The magnitude of the rotor flux vector is regulated at the reference nominal value.

The system of six-phase induction motor with DFOC and Fuzzy-Logic control can be applied where high reliability and accuracy are required.